

- (a) If  $\alpha = 30^\circ$  and  $v_0 = 800$  m/s, then the equations become  
 $x = (800 \cos 30^\circ)t = 400\sqrt{3}t$  and  $y = (800 \sin 30^\circ)t - \frac{1}{2}(9.8)t^2$   
 $= 400t - 4.9t^2$ .  $y = 0$  when  $t = 0$  (when the gun is fired) and again  
when  $t = \frac{400}{4.9} \approx 82$  s. Then  $x = (400\sqrt{3})\left(\frac{400}{4.9}\right) \approx 56557$  m, so the  
bullet hits the ground about 56 km from the gun. The formula for  $y$   
is quadratic in  $t$ . To find the maximum  $y$ -value, we will complete the  
square:

$$y = -4.9\left(t^2 - \frac{400}{4.9}t\right) = -4.9\left[t^2 - \frac{400}{4.9}t + \left(\frac{200}{4.9}\right)^2\right] + \frac{200^2}{4.9}$$

$$= -4.9\left(t - \frac{200}{4.9}\right)^2 + \frac{200^2}{4.9} \leq \frac{200^2}{4.9}$$

with equality when  $t = \frac{200}{4.9}$  s, so the maximum height attained is  
 $\frac{200^2}{4.9} \approx 8163$  m.

(b)  $x = (v_0 \cos \alpha)t \Rightarrow t = \frac{x}{v_0 \cos \alpha}$ .

$$y = (v_0 \sin \alpha)t - 4.9t^2 \Rightarrow y = (v_0 \sin \alpha) \frac{x}{v_0 \cos \alpha} - 4.9\left(\frac{x}{v_0 \cos \alpha}\right)^2 =$$

$$(\tan \alpha)x - \left(\frac{4.9}{v_0^2 \cos^2 \alpha}\right)x^2, \text{ which is the equation of a parabola (quadratic}$$

in  $x$ ).