

$$x \frac{dy}{dx} - 4y = x^2 e^x \Rightarrow y' - \frac{4}{x} y = x e^x.$$

$I(x) = e^{\int P(x) dx} = e^{\int (-4/x) dx} = e^{-4 \ln|x|} = (e^{\ln|x|})^{-4} = |x|^{-4} = x^{-4}$. Multiplying the differential equation by $I(x)$ gives $x^{-4} y' - 4x^{-5} y = x^{-3} e^x \Rightarrow (x^{-4} y)' = x^{-3} e^x \Rightarrow x^{-4} y = \int x^{-3} e^x dx \Rightarrow y = (\int x^{-3} e^x dx + C) x^4$.