

$\frac{1}{2} + \frac{1}{5} + \frac{1}{8} + \frac{1}{11} + \frac{1}{14} + \cdots = \sum_{n=1}^{\infty} \frac{1}{3n-1}$ . The function

$f(x) = \frac{1}{3x-1}$  is continuous, positive, and decreasing on  $[1, \infty)$ , so the Integral Test applies.

$$\begin{aligned} \int_1^{\infty} \frac{1}{3x-1} dx &= \lim_{t \rightarrow \infty} \int_1^t \frac{1}{3x-1} dx = \lim_{t \rightarrow \infty} \left[ \frac{1}{3} \ln |3x-1| \right]_1^t \\ &= \frac{1}{3} \lim_{t \rightarrow \infty} (\ln(3t-1) - \ln 2) = \infty, \end{aligned}$$

so the series  $\sum_{n=1}^{\infty} \frac{1}{3n-1}$  diverges.