Let the dimensions of the box be x, y, and z, so its volume is $f(x, y, z) = xyz$, its surface area is $2xy + 2yz + 2xz = 1500$ and its total edge length is $4x+4y+4z = 200$. We find the extreme values of $f(x, y, z)$ subject to the constraints $g(x, y, z) = xy+yz+xz = 750$ and $h(x, y, z) = x+y+z = 50$. Then

$$\begin{aligned} \nabla f &= \langle yz, xz, xy \rangle = \lambda \nabla g + \mu \nabla h \\ &= \langle \lambda(y+z), \lambda(x+z), \lambda(x+y) \rangle + \langle \mu, \mu, \mu \rangle . \end{aligned}$$

So $yz = \lambda(y + z) + \mu$ **(1)**, $xz = \lambda(x + z) + \mu$ **(2)**, and $xy = \lambda(x + y) + \mu$ **(3)**. Notice that the box can't be a cube or else $x = y = z = \frac{50}{3}$ but then $xy+yz+xz = \frac{2500}{3} \neq 750$. Assume x is the distinct side, that is, $x \neq y$, $x \neq z$. Then **(1)** minus **(2)** implies $z\,(y - x) = \lambda(y - x)$ or $\lambda = z$, and **(1)** minus **(3)** implies $y(z - x) = \lambda(z - x)$ or $\lambda = y$. So $y = z = \lambda$ and $x + y + z = 50$ implies $x = 50 - 2\lambda$; also $xy + yz + xz = 750$ implies $x(2\lambda) + \lambda^2 = 750$. Hence $50 - 2\lambda = \dfrac{750 - \lambda^2}{2\lambda}$ or $3\lambda^2 - 100\lambda + 750 = 0$ and $\lambda = \dfrac{50 \pm 5\sqrt{10}}{3}$, giving the points $\left(\frac{1}{3}\left(50 \mp 10\sqrt{10}\right), \frac{1}{3}\left(50 \pm 5\sqrt{10}\right), \frac{1}{3}\left(50 \pm 5\sqrt{10}\right)\right)$. Thus the minimum of f is
$f\left(\frac{1}{3}\left(50 - 10\sqrt{3}\right), \frac{1}{3}\left(50 + 5\sqrt{10}\right), \frac{1}{3}\left(50 + 5\sqrt{10}\right)\right) = \frac{1}{27}\left(87{,}500 - 2500\sqrt{10}\right)$,
and its maximum is
$f\left(\frac{1}{3}\left(50 + 10\sqrt{10}\right), \frac{1}{3}\left(50 - 5\sqrt{10}\right), \frac{1}{3}\left(50 - 5\sqrt{10}\right)\right) = \frac{1}{27}\left(87{,}500 + 2500\sqrt{10}\right)$.
Note: If either y or z is the distinct side, then symmetry gives the same result.