

The paraboloid  $z = 10 + 2x^2 + 2y^2$  intersects the plane  $z = 16$  when  $16 = 10 + 2x^2 + 2y^2$  or  $x^2 + y^2 = 3$  and we are restricted to the first octant, so

$$\begin{aligned} V &= \iint_{\substack{x^2+y^2 \leq 3, \\ x \geq 0, y \geq 0}} [16 - (10 + 2x^2 + 2y^2)] dA = \int_0^{\pi/2} \int_0^{\sqrt{3}} [16 - (10 + 2r^2)] r dr d\theta \\ &= \int_0^{\pi/2} d\theta \int_0^{\sqrt{3}} (6r - 2r^3) dr = [\theta]_0^{\pi/2} [3r^2 - \frac{1}{2}r^4]_0^{\sqrt{3}} = \frac{\pi}{2} \cdot \frac{9}{2} = \frac{9}{4}\pi \end{aligned}$$