

First let $u = e^{-\theta}$, $dv = \cos 7\theta d\theta \Rightarrow du = -e^{-\theta} d\theta$, $v = \frac{1}{7} \sin 7\theta$.

Then $I = \int e^{-\theta} \cos 7\theta d\theta = \frac{1}{7}e^{-\theta} \sin 7\theta - \int \frac{1}{7} \sin 7\theta (-e^{-\theta} d\theta)$

$= \frac{1}{7}e^{-\theta} \sin 7\theta + \frac{1}{7} \int e^{-\theta} \sin 7\theta d\theta$. Next let $U = e^{-\theta}$, $dV = \sin 7\theta d\theta$

$\Rightarrow dU = -e^{-\theta} d\theta$, $V = -\frac{1}{7} \cos 7\theta$, so $\int e^{-\theta} \sin 7\theta d\theta$

$= -\frac{1}{7}e^{-\theta} \cos 7\theta - \int (-\frac{1}{7}) \cos 7\theta (-e^{-\theta} d\theta) = -\frac{1}{7}e^{-\theta} \cos 7\theta - \frac{1}{7} \int e^{-\theta} \cos 7\theta d\theta$.

So $I = \frac{1}{7}e^{-\theta} \sin 7\theta + \frac{1}{7} [(-\frac{1}{7}e^{-\theta} \cos 7\theta) - \frac{1}{7}I]$

$= \frac{1}{7}e^{-\theta} \sin 7\theta - \frac{1}{49}e^{-\theta} \cos 7\theta - \frac{1}{49}I \Rightarrow$

$\frac{50}{49}I = \frac{1}{7}e^{-\theta} \sin 7\theta - \frac{1}{49}e^{-\theta} \cos 7\theta + C_1 \Rightarrow$

$I = \frac{49}{50}(\frac{1}{7}e^{-\theta} \sin 7\theta - \frac{1}{49}e^{-\theta} \cos 7\theta + C_1)$

$= \frac{7}{50}e^{-\theta} \sin 7\theta - \frac{1}{50}e^{-\theta} \cos 7\theta + C$.