If $a_n = \frac{n}{6^n}(x+6)^n$, then $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)(x+6)^{n+1}}{6^{n+1}} \cdot \frac{6^n}{n(x+6)^n}\right|$ $= \frac{|x+6|}{6}\lim_{n\to\infty}\frac{n+1}{n} = \frac{|x+6|}{6}$.

By the Ratio Test, the series $\sum_{n=1}^{\infty}\frac{n}{6^n}(x+6)^n$ converges when $\frac{|x+6|}{6} < 1 \quad \Leftrightarrow$ $|x+6| < 6 \quad [R = 6\,] \quad \Leftrightarrow \quad -6 < x+6 < 6 \quad \Leftrightarrow \quad -12 < x < 0$. When $x = -12$ or 0, both series $\sum_{n=1}^{\infty}(\mp 1)^n\, n$ diverge by the Test for Divergence since $\lim_{n\to\infty}|(\mp 1)^n\, n| = \infty$. Thus, the interval of convergence is $I = (-12, 0)$.