

$$f(x, y) = 3 + 6x\sqrt{y} \Rightarrow \nabla f(x, y) = \langle 6\sqrt{y}, 6x \cdot \frac{1}{2}y^{-1/2} \rangle = \langle 6\sqrt{y}, 3x/\sqrt{y} \rangle,$$

$\nabla f(3, 4) = \langle 12, \frac{9}{2} \rangle$, and a unit vector in the direction of \mathbf{v} is

$$\mathbf{u} = \frac{1}{\sqrt{(-4)^2 + (-3)^2}} \langle -4, -3 \rangle = \langle -\frac{4}{5}, -\frac{3}{5} \rangle, \text{ so}$$

$$D_{\mathbf{u}} f(3, 4) = \nabla f(3, 4) \cdot \mathbf{u} = \langle 12, \frac{9}{2} \rangle \cdot \langle -\frac{4}{5}, -\frac{3}{5} \rangle = -\frac{123}{10}.$$