$$\begin{split} f(x,y) &= 8 - 2x + 4y - x^2 - 4y^2 \implies f_x = -2 - 2x , f_y = 4 - 8y , \\ f_{xx} &= -2 , \quad f_{xy} = 0 , f_{yy} = -8 . \text{ Then } f_x = 0 \text{ and } f_y = 0 \text{ imply} \\ x &= -1 \text{ and } y = \frac{1}{2} \text{ , and the only critical point is } \left(-1, \frac{1}{2}\right) . \\ D(x,y) &= f_{xx} f_{yy} - \left(f_{xy}\right)^2 = (-2)(-8) - 0^2 = 16 \text{ , and since} \\ D\left(-1, \frac{1}{2}\right) = 16 > 0 \text{ and } f_{xx}\left(-1, \frac{1}{2}\right) = -2 < 0 , f\left(-1, \frac{1}{2}\right) = 10 \text{ is a local maximum by the Second Derivatives Test.} \end{split}$$

