

$f(x, y) = 4x^2 + 4y^2$ ,  $g(x, y) = xy = 1$ , and  $\nabla f = \lambda \nabla g \Rightarrow$   
 $\langle 8x, 8y \rangle = \langle \lambda y, \lambda x \rangle$ , so  $8x = \lambda y$ ,  $8y = \lambda x$ , and  $xy = 1$ . From the last  
equation,  $x \neq 0$  and  $y \neq 0$ , so  $8x = \lambda y \Rightarrow \lambda = 8x/y$ . Substituting,  
we have  $8y = (8x/y)x \Rightarrow y^2 = x^2 \Rightarrow y = \pm x$ . But  $xy = 1$ , so  
 $x = y = \pm 1$  and the possible points for the extreme values of  $f$  are  $(1, 1)$   
and  $(-1, -1)$ . Here there is no maximum value, since the constraint  $xy = 1$   
allows  $x$  or  $y$  to become arbitrarily large, and hence  $f(x, y) = 4x^2 + 4y^2$  can  
be made arbitrarily large. The minimum value is  $f(1, 1) = f(-1, -1) = 8$ .