$\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 5 & 1 \\ 1 & 5 \end{vmatrix} = 24$ and $x - 4y = (5u+v) - 4(u+5v) = u - 19v$.

To find the region S in the uv-plane that corresponds to R we first find the corresponding boundary under the given transformation.

The line through $(0,0)$ and $(5,1)$ is $y = \frac{1}{5}x$ which is the image of $u+5v = \frac{1}{5}(5u+v) \quad \Rightarrow \quad v = 0$; the line through $(5,1)$ and $(1,5)$ is $x+y = 6$ which is the image of $(5u+v)+(u+5v) = 6 \quad \Rightarrow$ $u+v = 1$; the line through $(0,0)$ and $(1,5)$ is $y = 5x$ which is the image of $u+5v = 5(5u+v) \quad \Rightarrow \quad u = 0$.

Thus S is the triangle $0 \le v \le 1-u$, $0 \le u \le 1$ in the uv-plane and

$$\begin{aligned}
\iint_R (x-4y)\,dA &= \int_0^1 \int_0^{1-u} (u-19v)\,|24|\;dv\,du \\
&= 24 \int_0^1 \left[uv - (19/2)v^2\right]_{v=0}^{v=1-u}\;du \\
&= 24 \int_0^1 \left((u-u^2) - (19/2)(1-u)^2\right)\,du \\
&= 24\left[(1/2)u^2 + (-1/3)u^3 + (19/6)(1-u)^3\right]_0^1 \\
&= 24(1/2 + -1/3 - 19/6) = -72
\end{aligned}$$