

(a) $V = \ell wh$, so by the Chain Rule,

$$\begin{aligned}\frac{dV}{dt} &= \frac{\partial V}{\partial \ell} \frac{d\ell}{dt} + \frac{\partial V}{\partial w} \frac{dw}{dt} + \frac{\partial V}{\partial h} \frac{dh}{dt} = wh \frac{d\ell}{dt} + \ell h \frac{dw}{dt} + \ell w \frac{dh}{dt} \\ &= 8 \cdot 8 \cdot 6 + 2 \cdot 8 \cdot 6 + 2 \cdot 8 \cdot (-5) = 400 \text{ m}^3/\text{s}.\end{aligned}$$

(b) $S = 2(\ell w + \ell h + wh)$, so by the Chain Rule,

$$\begin{aligned}\frac{dS}{dt} &= \frac{\partial S}{\partial \ell} \frac{d\ell}{dt} + \frac{\partial S}{\partial w} \frac{dw}{dt} + \frac{\partial S}{\partial h} \frac{dh}{dt} = 2(w + h) \frac{d\ell}{dt} + 2(\ell + h) \frac{dw}{dt} + 2(\ell + w) \frac{dh}{dt} \\ &= 2(8 + 8)6 + 2(2 + 8)6 + 2(2 + 8)(-5) = 212 \text{ m}^2/\text{s}\end{aligned}$$

(c) $L^2 = \ell^2 + w^2 + h^2 \Rightarrow$

$$\begin{aligned}2L \frac{dL}{dt} &= 2\ell \frac{d\ell}{dt} + 2w \frac{dw}{dt} + 2h \frac{dh}{dt} = 2(2)(6) + 2(8)(6) + 2(8)(-5) = 40 \Rightarrow \\ dL/dt &= \frac{40}{2\sqrt{132}} \approx 1.74 \text{ m/s}.\end{aligned}$$