$$\begin{aligned}\lim_{x\to 0}\frac{1-\cos(5x)}{1+5x-e^{5x}} \qquad &= \lim_{x\to 0}\frac{1-\left(1-\frac{1}{2!}(5x)^2+\frac{1}{4!}(5x)^4-\cdots\right)}{1+5x-\left(1+5x+\frac{1}{2!}(5x)^2+\frac{1}{3!}(5x)^3+\frac{1}{4!}(5x)^4+\cdots\right)}\\ &= \lim_{x\to 0}\frac{\frac{1}{2!}(5x)^2-\frac{1}{4!}(5x)^4+\cdots}{-\frac{1}{2!}(5x)^2-\frac{1}{3!}(5x)^3-\frac{1}{4!}(5x)^4-\cdots}\\ &= \lim_{x\to 0}\frac{\frac{25}{2!}-\frac{625}{4!}x^2+\cdots}{-\frac{25}{2!}-\frac{125}{3!}x-\frac{625}{4!}x^2-\cdots} = \frac{\frac{25}{2}-0}{-\frac{25}{2}-0} = -1\end{aligned}$$

since power series are continuous functions.