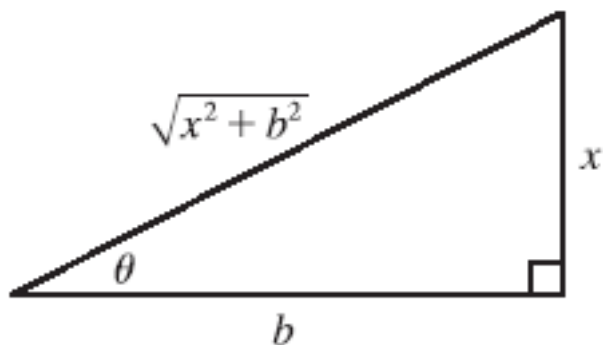


Let  $x = 9 \tan \theta$ , where  $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ . Then  $dx = 9 \sec^2 \theta d\theta$  and

$$\begin{aligned}\sqrt{x^2 + 81} &= \sqrt{81 \tan^2 \theta + 81} = \sqrt{81(\tan^2 \theta + 1)} = \sqrt{81 \sec^2 \theta} \\ &= 9 |\sec \theta| = 9 \sec \theta \text{ for the relevant values of } \theta.\end{aligned}$$

$$\begin{aligned}\int \frac{x^3}{\sqrt{x^2 + 81}} dx &= \int \frac{729 \tan^3 \theta}{9 \sec \theta} 9 \sec^2 \theta d\theta = 729 \int \tan^3 \theta \sec \theta d\theta \\ &= 729 \int \tan^2 \theta \tan \theta \sec \theta d\theta \\ &= 729 \int (\sec^2 \theta - 1) \tan \theta \sec \theta d\theta \\ &= 729 \int (u^2 - 1) du \quad [u = \sec \theta, du = \sec \theta \tan \theta d\theta] \\ &= 729 \left( \frac{1}{3} u^3 - u \right) + C = 729 \left( \frac{1}{3} \sec^3 \theta - \sec \theta \right) + C \\ &= 729 \left[ \frac{1}{3} \frac{(x^2 + 81)^{3/2}}{729} - \frac{\sqrt{x^2 + 81}}{9} \right] + C \\ &= \frac{1}{3} (x^2 + 81)^{3/2} - 81 \sqrt{x^2 + 81} + C\end{aligned}$$



$$b = 9$$