$z = f(x, y) = y^2 - x^2$ with $4 \le x^2 + y^2 \le 16$. Then

$$A(S) = \iint_D \sqrt{1 + 4x^2 + 4y^2}\, dA = \int_0^{2\pi} \int_2^4 \sqrt{1 + 4r^2}\, r\, dr\, d\theta$$
$$= \int_0^{2\pi} d\theta \int_2^4 r\sqrt{1 + 4r^2}\, dr = \left[\, \theta \,\right]_0^{2\pi} \left[\tfrac{1}{12}(1 + 4r^2)^{3/2} \right]_2^4 = \tfrac{\pi}{6} \left(65^{3/2} - 17^{3/2} \right)$$