

A cross-section is a disk with radius $4 - \frac{1}{2}x$, so its area is

$$A(x) = \pi\left(4 - \frac{1}{2}x\right)^2.$$

$$\begin{aligned} V &= \int_0^1 A(x) dx = \int_0^1 \pi\left(4 - \frac{1}{2}x\right)^2 dx \\ &= \pi \int_0^1 \left(16 - 4x + \frac{1}{4}x^2\right) dx \\ &= \pi \left[16x - 2x^2 + \frac{1}{12}x^3\right]_0^1 \\ &= \pi \left[\left(16 - 2 + \frac{1}{12}\right) - (0 - 0 + 0)\right] \\ &= \frac{169}{12}\pi \end{aligned}$$



