

Using y and z as parameters, we have $\mathbf{r}(y, z) = (y + 4z^2)\mathbf{i} + y\mathbf{j} + z\mathbf{k}$,
 $0 \leq y \leq 1$, $0 \leq z \leq 1$.

Then $\mathbf{r}_y \times \mathbf{r}_z = (\mathbf{i} + \mathbf{j}) \times (8z\mathbf{i} + \mathbf{k}) = \mathbf{i} - \mathbf{j} - 8z\mathbf{k}$

and $|\mathbf{r}_y \times \mathbf{r}_z| = \sqrt{2 + 64z^2}$. Thus

$$\begin{aligned} \iint_S z \, dS &= \int_0^1 \int_0^1 z \sqrt{2 + 64z^2} \, dy \, dz = \int_0^1 z \sqrt{2 + 64z^2} \, dz \\ &= \left[\frac{1}{128} \cdot \frac{2}{3} (2 + 64z^2)^{3/2} \right]_0^1 = \frac{1}{192} (66^{3/2} - 2^{3/2}). \end{aligned}$$