

$\operatorname{div}\mathbf{F} = 2x + x + 1 = 3x + 1$ so

$$\begin{aligned}\iint\iint_E \operatorname{div}\mathbf{F} \, dV &= \iint\iint_E (3x + 1) \, dV = \int_0^{2\pi} \int_0^3 \int_0^{9-r^2} (3r \cos \theta + 1) r \, dz \, dr \, d\theta \\ &= \int_0^3 \int_0^{2\pi} r(3r \cos \theta + 1)(9 - r^2) \, d\theta \, dr \\ &= \int_0^3 r(9 - r^2) \left[3r \sin \theta + \theta \right]_{\theta=0}^{\theta=2\pi} \, dr \\ &= 2\pi \int_0^3 (9r - r^3) \, dr = 2\pi \left[\frac{9}{2}r^2 - \frac{1}{4}r^4 \right]_0^3 \\ &= 2\pi \left(\frac{81}{2} - \frac{81}{4} \right) = \frac{81}{2}\pi\end{aligned}$$

On S_1 : The surface is $z = 9 - x^2 - y^2$, $x^2 + y^2 \leq 9$, with upward orientation, and $\mathbf{F} = x^2\mathbf{i} + xy\mathbf{j} + (9 - x^2 - y^2)\mathbf{k}$. Then

$$\begin{aligned}\iint_{S_1} \mathbf{F} \cdot d\mathbf{S} &= \iint_D [-(x^2)(-2x) - (-xy)(-2y) + (9 - x^2 - y^2)] \, dA \\ &= \iint_D [2x(x^2 + y^2) + 9 - (x^2 + y^2)] \, dA \\ &= \int_0^{2\pi} \int_0^3 (2r \cos \theta \cdot r^2 + 9 - r^2) r \, dr \, d\theta \\ &= \int_0^{2\pi} \left[\frac{2}{5}r^5 \cos \theta + \frac{9}{2}r^2 - \frac{1}{4}r^4 \right]_{r=0}^{r=3} \, d\theta \\ &= \int_0^{2\pi} \left(\frac{486}{2} \cos \theta + \frac{81}{4} \right) \, d\theta = \left[\frac{486}{2} \sin \theta + \frac{81}{4} \theta \right]_0^{2\pi} = \frac{81}{2}\pi\end{aligned}$$

On S_2 : The surface is $z = 0$ with downward orientation, so

$$\mathbf{F} = x^2\mathbf{i} + xy\mathbf{j}, \mathbf{n} = -\mathbf{k} \text{ and } \iint_{S_2} \mathbf{F} \cdot \mathbf{n} \, dS = \iint_{S_2} 0 \, dS = 0.$$

Thus $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_{S_1} \mathbf{F} \cdot d\mathbf{S} + \iint_{S_2} \mathbf{F} \cdot d\mathbf{S} = \frac{81}{2}\pi$.

