

The function $f(x) = \frac{5}{\sqrt[5]{x}} = 5x^{-1/5}$ is continuous, positive, and decreasing on $[1, \infty)$, so the Integral Test applies.

$$\begin{aligned} \int_1^{\infty} 5x^{-1/5} dx &= \lim_{t \rightarrow \infty} \int_1^t 5x^{-1/5} dx = \lim_{t \rightarrow \infty} 5 \left[\frac{5}{4} x^{4/5} \right]_1^t = \lim_{t \rightarrow \infty} 5 \left(\frac{5}{4} t^{4/5} - \frac{5}{4} \right) \\ &= \infty, \text{ so } \sum_{n=1}^{\infty} \frac{5}{\sqrt[5]{n}} \text{ diverges.} \end{aligned}$$