$\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 1/v & -u/v^2 \\ 0 & 1 \end{vmatrix} = \dfrac{1}{v}$, $xy = u$, $y = x$ is the image of the parabola $v^2 = u$, $y = \frac{5}{4}x$ is the image of the parabola $v^2 = \frac{5}{4}u$, and the hyperbolas $xy = 1$, $xy = \frac{5}{4}$ are the images of the lines $u = 1$ and $u = \frac{5}{4}$ respectively. Thus,

$$\begin{aligned} \iint_R 6xy\,dA &= 6\int_1^{5/4}\int_{\sqrt{u}}^{\sqrt{(5/4)u}} u\left(\frac{1}{v}\right) dv\,du = 6\int_1^{5/4} u\left(\ln(\sqrt{\tfrac{5}{4}u}) - \ln(\sqrt{u})\right) du \\ &= 6\int_1^{5/4} u\ln(\sqrt{5/4})\,du = \tfrac{27}{16}\ln(\sqrt{5/4}) = \tfrac{27}{32}\ln(5/4). \end{aligned}$$