The function $f(x) = 1/\sqrt{x+2} = (x+2)^{-1/2}$ is continuous, positive, and decreasing on $[1, \infty)$, so the Integral Test applies.
$\int_1^\infty (x+2)^{-1/2}\,dx = \lim\limits_{t\to\infty} \int_1^t (x+2)^{-1/2}\,dx = \lim\limits_{t\to\infty} \left[2(x+2)^{1/2}\right]_1^t$
$= \lim\limits_{t\to\infty} \left(2\sqrt{t+2} - 2\sqrt{3}\right) = \infty$, so the series $\sum\limits_{n=1}^{\infty} 1/\sqrt{n+2}$ diverges.