

$$xy' - 2y = 9x^2 \quad [\text{divide by } x] \Rightarrow y' + \left(-\frac{2}{x}\right)y = 9x \quad (\star).$$

$$I(x) = e^{\int P(x) dx} = e^{\int (-2/x) dx} = e^{-2 \ln|x|} = e^{\ln|x|^{-2}} = e^{\ln(1/x^2)} = 1/x^2.$$

Multiplying the differential equation (\star) by $I(x)$ gives

$$\begin{aligned} \frac{1}{x^2} y' - \frac{2}{x^3} y &= \frac{9}{x} \Rightarrow \left(\frac{1}{x^2} y\right)' = \frac{9}{x} \Rightarrow \frac{1}{x^2} y = 9 \ln|x| + C \Rightarrow \\ y &= x^2(9 \ln|x| + C) = 9x^2 \ln|x| + Cx^2. \end{aligned}$$