

$$\begin{aligned}\frac{dz}{dt} + 6e^{t+z} &= 0 \Rightarrow \frac{dz}{dt} = -6e^t e^z \Rightarrow \int e^{-z} dz = -6 \int e^t dt \Rightarrow -e^{-z} = \\ -6e^t + C &\Rightarrow e^{-z} = 6e^t - C \Rightarrow \frac{1}{e^z} = 6e^t - C \Rightarrow \\ e^z &= \frac{1}{6e^t - C} \Rightarrow z = \ln\left(\frac{1}{6e^t - C}\right) \Rightarrow z = -\ln(6e^t - C)\end{aligned}$$