

The region of integration is given in spherical coordinates by

$E = \{(\rho, \theta, \phi) | 0 \leq \rho \leq 2, 0 \leq \theta \leq 3\pi/2, 0 \leq \phi \leq \pi/4\}$ . This represents the solid region in the octants  $\{1, 2, 3\}$  bounded above by the sphere  $\rho = 2$  and below by the cone  $\phi = \pi/4$ .

$$\begin{aligned} \int_0^{\pi/4} \int_0^{3\pi/2} \int_0^2 \rho^2 \sin(\phi) d\rho d\theta d\phi &= \int_0^{\pi/4} \sin(\phi) d\phi \int_0^{3\pi/2} d\theta \int_0^2 \rho^2 d\rho \\ &= [-\cos(\phi)]_0^{\pi/4} [\theta]_0^{3\pi/2} \left[\frac{1}{3}\rho^3\right]_0^2 \\ &= \left(1 - \frac{\sqrt{2}}{2}\right) \left(\frac{3\pi}{2}\right) (8/3) = (2)(2 - \sqrt{2})\pi \end{aligned}$$