

$$\begin{aligned}
f(x) &= \frac{3+x}{1-x} = (3+x) \left( \frac{1}{1-x} \right) = (3+x) \sum_{n=0}^{\infty} x^n = \sum_{n=0}^{\infty} (3+x)x^n \quad . \\
&= 3 \sum_{n=0}^{\infty} x^n + \sum_{n=0}^{\infty} x^{n+1} = 3 + \sum_{n=1}^{\infty} 3x^n + \sum_{n=1}^{\infty} x^n = 3 + 4 \sum_{n=1}^{\infty} x^n
\end{aligned}$$

The series converges when  $|x| < 1$ , so  $R = 1$  and  $I = (-1, 1)$ .

*A second approach:*

$$\begin{aligned}
f(x) &= \frac{3+x}{1-x} = \frac{-(1-x) + 4}{1-x} = -1 + 4 \left( \frac{1}{1-x} \right) \quad . \\
&= -1 + 4 \sum_{n=0}^{\infty} x^n = 3 + 4 \sum_{n=1}^{\infty} x^n
\end{aligned}$$

*A third approach:*

$$\begin{aligned}
f(x) &= \frac{3+x}{1-x} = (3+x) \left( \frac{1}{1-x} \right) = (3+x)(1+x+x^2+x^3+\dots) \\
&= (3+3x+3x^2+3x^3+\dots) + (x+x^2+x^3+x^4+\dots) \\
&= 3+4x+4x^2+4x^3+\dots = 3+4 \sum_{n=1}^{\infty} x^n.
\end{aligned}$$