

$$x^2 y' + 2xy = 15 \cos^2(x) \Rightarrow y' + \frac{2}{x} y = \frac{15 \cos^2(x)}{x^2}. \quad I(x) = e^{\int P(x) dx} = e^{\int 2/x dx} = e^{2 \ln|x|} = e^{\ln(x^2)} = x^2.$$

Multiplying by $I(x)$ gives us our original equation back. You may have noticed this immediately, since $P(x)$ is the derivative of the coefficient of y' .

We rewrite it as $(x^2 y)' = 15 \cos^2(x)$. Thus,

$$x^2 y = \int 15 \cos^2(x) dx = \int \frac{15}{2} (1 + \cos(2x)) dx = \frac{15}{2} x + \frac{15}{4} \sin(2x) + C \Rightarrow y = \frac{15}{2x} + \frac{15}{4x^2} \sin(2x) + \frac{C}{x^2} \quad \text{or} \quad y = \frac{15}{2x} + \frac{15}{2x^2} \sin(x) \cos(x) + \frac{C}{x^2}.$$