

$$\frac{7x^2 + 2x - 7}{x^3 - x} = \frac{7x^2 + 2x - 7}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}. \text{ Multiply both sides}$$

by $x(x+1)(x-1)$ to get

$7x^2 + 2x - 7 = A(x+1)(x-1) + Bx(x-1) + Cx(x+1)$. Substituting 0 for x gives $-7 = -A \Leftrightarrow A = 7$. Substituting -1 for x gives $-2 = 2B \Leftrightarrow B = -1$. Substituting 1 for x gives $2 = 2C \Leftrightarrow C = 1$. Thus,

$$\begin{aligned} \int \frac{7x^2 + 2x - 7}{x^3 - x} dx &= \int \left(\frac{7}{x} - \frac{1}{x+1} + \frac{1}{x-1} \right) dx \\ &= 7 \ln |x| - \ln |x+1| + \ln |x-1| + C = \ln \left| \frac{x^7(x-1)}{(x+1)} \right| + C. \end{aligned}$$