

Let d be the distance from $(3, 6, -4)$ to any point (x, y, z) on the plane $x + y - z = 4$, so $d = \sqrt{(x - 3)^2 + (y - 6)^2 + (z - (-4))^2}$ where $z = x + y - 4$, and we minimize $d^2 = f(x, y) = (x - 3)^2 + (y - 6)^2 + (x + y)^2$. Then $f_x(x, y) = 2(x - 3) + 2(x + y) = 4x + 2y - 6$, $f_y(x, y) = 2(y - 6) + 2(x + y) = 2x + 4y - 12$. Solving $4x + 2y - 6 = 0$ and $2x + 4y - 12 = 0$ simultaneously gives $x = 0$, $y = 3$. An absolute minimum exists (since there is a minimum distance from the point to the plane) and it must occur at a critical point, so the shortest distance occurs for $x = 0$, $y = 3$ for which $d = \sqrt{(0 - 3)^2 + (3 - 6)^2 + (0 + 3)^2} = \frac{9}{\sqrt{3}}$.