If $a_n = \dfrac{(x-5)^n}{n^3+1}$, then $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty}\left|\dfrac{(x-5)^{n+1}}{(n+1)^3+1}\cdot\dfrac{n^3+1}{(x-5)^n}\right|$ $= |x-5|\lim\limits_{n\to\infty}\dfrac{n^3+1}{(n+1)^3+1} = |x-5|$.

By the Ratio Test, the series $\sum\limits_{n=0}^{\infty}\dfrac{(x-5)^n}{n^3+1}$ converges when $|x-5|<1$ $[R = 1\,]$ $\Leftrightarrow$ $-1 < x-5 < 1$ $\Leftrightarrow$ $4 < x < 6$. When $x = 4$, the series $\sum\limits_{n=0}^{\infty}(-1)^n\dfrac{1}{n^3+1}$ converges by the Alternating Series Test; when $x = 6$, the series $\sum\limits_{n=0}^{\infty}\dfrac{1}{n^3+1}$ converges by comparison with the p-series $\sum\limits_{n=1}^{\infty}\dfrac{1}{n^3}$ $[p = 3 > 1\,]$. Thus, the interval of convergence is $I = [4, 6]$.