

$xyz = \tan(x + y + z)$. Let $F(x, y, z) = xyz - \tan(x + y + z) = 0$, so

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{yz - \sec^{-2}(x + y + z)}{xy - \sec^{-2}(x + y + z)} \quad \text{and}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{xz - \sec^{-2}(x + y + z)}{xy - \sec^{-2}(x + y + z)}.$$