

$$\begin{aligned}\operatorname{curl}\mathbf{F} &= \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^z & 9 & xe^z \end{vmatrix} \\ &= (0 - 0)\mathbf{i} - (e^z - e^z)\mathbf{j} + (0 - 0)\mathbf{k} = \mathbf{0}\end{aligned}$$

\mathbf{F} is defined on all of \mathbb{R}^3 with component functions that have continuous partial derivatives, so \mathbf{F} is conservative. Thus there exists a function f such that $\nabla f = \mathbf{F}$. Then $f_x(x, y, z) = e^z$ implies $f(x, y, z) = xe^z + g(y, z) \Rightarrow f_y(x, y, z) = g_y(y, z)$. But $f_y(x, y, z) = 9$, so $g(y, z) = 9y + h(z)$ and $f(x, y, z) = xe^z + 9y + h(z)$. Thus $f_z(x, y, z) = xe^z + h'(z)$ but $f_z(x, y, z) = xe^z$, so $h(z) = K$, a constant. Hence a potential function for \mathbf{F} is $f(x, y, z) = xe^z + 9y + K$.