

$$\sum_{n=1}^{\infty} \frac{1+9^n}{4^n} = \sum_{n=1}^{\infty} \left(\frac{1}{4^n} + \frac{9^n}{4^n} \right) = \sum_{n=1}^{\infty} \left[\left(\frac{1}{4} \right)^n + \left(\frac{9}{4} \right)^n \right] = \sum_{n=1}^{\infty} \left(\frac{1}{4} \right)^n + \sum_{n=1}^{\infty} \left(\frac{9}{4} \right)^n.$$

The first series is a convergent geometric series ($|r| = \frac{1}{4} < 1$), but the second series is a divergent geometric series ($|r| = \frac{9}{4} \geq 1$), so the original series is divergent.