

The distance from the origin to a point (x, y, z) on the surface is $d = \sqrt{x^2 + y^2 + z^2}$ where $y^2 = 4 + xz$, so we minimize $d^2 = x^2 + 4 + xz + z^2 = f(x, z)$. Then $f_x = 2x + z$, $f_z = x + 2z$, and $f_x = 0$, $f_z = 0 \Rightarrow x = 0$, $z = 0$, so the only critical point is $(0, 0)$. $D(0, 0) = (2)(2) - 1 = 3 > 0$ with $f_{xx}(0, 0) = 2 > 0$, so this is a minimum. Thus $y^2 = 4 + 0 \Rightarrow y = \pm 2$ and the points on the surface closest to the origin are $(0, \pm 2, 0)$.