

The sphere $x^2 + y^2 + z^2 = 16$ intersects the xy -plane in the circle $x^2 + y^2 = 16$, so

$$\begin{aligned} V &= 2 \iint_{1 \leq x^2 + y^2 \leq 16} \sqrt{16 - x^2 - y^2} \, dA \quad [\text{by symmetry}] = 2 \int_0^{2\pi} \int_1^4 \sqrt{16 - r^2} \, r \, dr \, d\theta \\ &= 2 \int_0^{2\pi} d\theta \int_1^4 r(16 - r^2)^{1/2} \, dr = 2[\theta]_0^{2\pi} \left[-\frac{1}{3}(16 - r^2)^{3/2}\right]_1^4 = -\frac{2}{3}(2\pi)(0 - 15^{3/2}) = \frac{4\pi}{3}(15\sqrt{15}) \\ &= 20 \cdot \sqrt{15}\pi \end{aligned}$$