If $a_n = (-1)^n \dfrac{(x-6)^n}{6n+1}$, then $\lim\limits_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty} \left|\dfrac{(x-6)^{n+1}}{6n+7} \cdot \dfrac{6n+1}{(x-6)^n}\right|$ $= |x-6| \lim\limits_{n\to\infty} \dfrac{6n+1}{6n+7} = |x-6|$.

By the Ratio Test, the series $\sum\limits_{n=0}^{\infty} (-1)^n \dfrac{(x-6)^n}{6n+1}$ converges when $|x-6| < 1 \quad [R=1] \quad \Leftrightarrow \quad -1 < x-6 < 1 \quad \Leftrightarrow \quad 5 < x < 7$. When $x = 5$, the series $\sum\limits_{n=0}^{\infty} \dfrac{1}{6n+1}$ diverges by limit comparison with the harmonic series (or by the Integral Test); when $x = 7$, the series $\sum\limits_{n=0}^{\infty} (-1)^n \dfrac{1}{6n+1}$ converges by the Alternating Series Test. Thus, the interval of convergence is $I = (5, 7]$.