

Let $F(x, y, z) = x^2 - 4y^2 + z^2 + yz$. Then $x^2 - 4y^2 + z^2 + yz = 3$ is a level surface of F and $\nabla F(x, y, z) = \langle 2x, -8y + z, 2z + y \rangle$.

(a) $\nabla F(1, 1, -3) = \langle 2, -11, -5 \rangle$ is a normal vector for the tangent plane at $(1, 1, -3)$, so an equation of the tangent plane is $2(x - 1) - 11(y - 1) - 5(z + 3) = 0$ or $2x - 11y - 5z = 6$.

(b) The normal line has direction $\langle 2, -11, -5 \rangle$, so parametric equations are $x = 1 + 2t$, $y = 1 - 11t$, $z = -3 - 5t$, and symmetric equations are $\frac{x - 1}{2} = \frac{y - 1}{-11} = \frac{z + 3}{-5}$.