(a) $f_x(x,y,z) = e^y$ implies $f(x,y,z) = xe^y + g(y,z)$ and so $f_y(x,y,z) = xe^y + g_y(y,z)$. But $f_y(x,y,z) = xe^y$ so $g_y(y,z) = 0 \quad \Rightarrow \quad g(y,z) = h(z)$. Thus $f(x,y,z) = xe^y + h(z)$ and $f_z(x,y,z) = 0 + h'(z)$. But $f_z(x,y,z) = (z+1)e^z$, so $h'(z) = (z+1)e^z \quad \Rightarrow \quad h(z) = ze^z + K$ (using integration by parts). Hence $f(x,y,z) = xe^y + ze^z$ (taking $K = 0$).

(b) $\mathbf{r}(0) = \langle 0,0,0 \rangle$, $\mathbf{r}(1) = \langle 2,1,1 \rangle$ so $\int_C \mathbf{F} \cdot d\mathbf{r} = f(2,1,1) - f(0,0,0) = 3e - 0 = 3e$.