

$\mathbf{F}(x, y) = \left\langle y - \ln(x^2 + y^2), 2 \arctan\left(\frac{y}{x}\right) \right\rangle$  and the region  $D$  enclosed by  $C$  is the disk with radius 4 centered at (3, 4).  $C$  is oriented positively, so

$$\begin{aligned} \int_C \mathbf{F} \cdot d\mathbf{r} &= \int_C (y - \ln(x^2 + y^2)) dx + \left(2 \arctan\left(\frac{y}{x}\right)\right) dy \\ &= \iint_D \left[ \frac{\partial}{\partial x} \left(2 \arctan\left(\frac{y}{x}\right)\right) - \frac{\partial}{\partial y} (y - \ln(x^2 + y^2)) \right] dA \\ &= \iint_D \left[ 2 \left( \frac{-yx^{-2}}{1 + (y/x)^2} \right) - \left( 1 - \frac{2y}{x^2 + y^2} \right) \right] dA \\ &= \iint_D \left[ -\frac{2y}{x^2 + y^2} - 1 + \frac{2y}{x^2 + y^2} \right] dA \\ &= - \iint_D dA = -(\text{area of } D) = -16\pi \end{aligned}$$