$(1+6t)\,\dfrac{du}{dt}+6u=1+6t, \quad t>0 \quad [\text{divide by } 1+6t] \quad \Rightarrow \quad \dfrac{du}{dt}+\dfrac{6}{1+6t}\,u=1 \quad (\star)$, which has the form $u'+P(t)\,u=Q(t)$. The integrating factor is $I(t)=e^{\int P(t)\,dt}=e^{\int[6/(1+6t)]\,dt}=e^{\ln(1+6t)}=1+6t$.

Multiplying $(\star)$ by $I(t)$ gives us our original equation back. We rewrite it as $[(1+6t)u]'=1+6t$. Thus, $(1+6t)u=\int(1+6t)\,dt=t+\frac{6}{2}t^2+C \quad \Rightarrow$ $u=\dfrac{t+\frac{6}{2}t^2+C}{1+6t}$ or $u=\dfrac{3t^2+t+2C}{(6t+1)}$.