

$$\frac{x^2 - 6x - 1}{(x - 1)^2(x^2 + 1)} = \frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{Cx + D}{x^2 + 1} \Rightarrow$$

$x^2 - 6x - 1 = A(x - 1)(x^2 + 1) + B(x^2 + 1) + (Cx + D)(x - 1)^2$ . Setting  $x = 1$  gives  $B = -3$ . Equating the coefficients of  $x^3$  gives  $A = -C$ . Equating the constant terms gives  $-1 = -A - 3 + D$ , so  $-2 = A - D$ , and setting  $x = 2$  gives  $A = 1$ . We have

$$\begin{aligned} \int \frac{x^2 - 6x - 1}{(x - 1)^2(x^2 + 1)} dx &= \int \left[ \frac{1}{x - 1} - \frac{3}{(x - 1)^2} - \frac{1x - 3}{x^2 + 1} \right] dx \\ &= \ln|x - 1| + \frac{3}{x - 1} - \frac{1}{2} \ln(x^2 + 1) + 3 \tan^{-1} x + C. \end{aligned}$$