$f(x) = xe^{-8x}$ is continuous and positive on $[1, \infty)$.
$f'(x) = -8xe^{-8x} + e^{-8x} = e^{-8x}(1-8x) < 0$ for $x > \frac{1}{8}$, so f is decreasing on $[1, \infty)$.
Thus, the Integral Test applies.

$$\begin{aligned} \int_1^\infty xe^{-8x}\,dx &= \lim_{b\to\infty} \int_1^b xe^{-8x}\,dx = \lim_{b\to\infty} \left[-\tfrac{1}{8}xe^{-8x} - \tfrac{1}{64}e^{-8x}\right]_1^b \quad \text{[by parts]} \\ &= \lim_{b\to\infty} \left[-\tfrac{1}{8}be^{-8b} - \tfrac{1}{64}e^{-8b} + \tfrac{1}{8}e^{-8} + \tfrac{1}{64}e^{-8}\right] = \frac{9}{64e^8} \end{aligned}$$

since $\lim\limits_{b\to\infty} \frac{1}{8}be^{-8b} = \lim\limits_{b\to\infty} \left(\frac{b}{8e^{8b}}\right) \overset{\text{H}}{=} \lim\limits_{b\to\infty} \left(\frac{1}{64e^{8b}}\right) = 0$ and $\lim\limits_{b\to\infty} e^{-8b} = 0$. Thus, $\sum\limits_{n=1}^{\infty} ne^{-8n}$ converges.