

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \Rightarrow e^{2x} = \sum_{n=0}^{\infty} \frac{(2x)^n}{n!} = \sum_{n=0}^{\infty} \frac{2^n x^n}{n!}, \text{ so } f(x) = 2e^x + e^{2x}$$
$$= \sum_{n=0}^{\infty} \frac{2}{n!} x^n + \sum_{n=0}^{\infty} \frac{2^n}{n!} x^n = \sum_{n=0}^{\infty} \frac{2^n + 2}{n!} x^n, R = \infty.$$